Answer Keys

Part I

1. 490

2. 657.8

3. 31

4. 289

5. 6350

6. 6868

7. 8

8. 111

9. 70

10. 18

11. 15840 ft.

12. 148

13. 54

14 $7\frac{2}{3}$

15. $\frac{55}{2}$.

16. 76

17. 2000

18. 16,761

19. 9408

20. 117

21. $72\frac{3}{16}$

22. 640

23. 4

24. 147°.

25. $2\frac{9}{40}$

26. $x = \sqrt{15}$

27. $12\frac{1}{2}$.

28. $26_{10.}$

29. 100π

30. 5

31. 180°.

32. $4\sqrt{3}$

33. 7.

34. 343.

35. $\frac{2}{3}$

36. 88

37. -3

38. 42

39. 72°

40. 1/3.

Part II:

41. 162 (minutes)
42. 18.
43. 250 (students)
44. 8 (times)
45. 420.
46. 132
47. 25 (sq units)
48. 0.
49. 34 (units)
50. 1/24
51. –1.
52. 0.
53. 24.
54. 22
55. 9.
56. 300 (degrees)
57. 14 (blue marbles).
58. 3 (sq units)
59. 7.
60. 100 (minutes)
61. 18 minutes.
62. 143.
63. 56 (triangles)
64. 300 (dollars)
65. 210.
66. 20 (games)
67. 54.
68. 1
69. 25.
70. $5\frac{3}{5}$.
71. 13/8.
72. 4
73. 100
74. 2/9.
75. 132 (units)
76. 16 (years)
77. 2/15.
78. 76 (integers)
79. 13.
80. 7.

Solutions to Part II:

41. Solution: 162 (minutes).
Method 1:
It takes 120 minutes from 10:37 a.m. to 12:37 p.m. There are 79 – 37 = 42 minutes from 12:37 to 1:19 . The answer is then 120 + 42 = 162 minutes.

Method 2:
It takes 180 minutes from 10:37 a.m. to 1:37 p.m . There are 37 – 19 = 18 minutes. The answer is then 1890 – 18 = 162 minutes.

42. Solution: 18.
$3x + 3 = 3x + 8 - 5 = 23 - 5 = 18$.

43. Solution: 250.
We know that three out of every five students at Gauss Middle School went to the Spring Fling. So two out of every five students did not go. Let x be the total number of students.
$\frac{2}{5} = \frac{100}{x} \quad \Rightarrow \quad x = 250.$

44. Solution: 8.
Let V_1 be the original volume and V_2 be the new volume.
$\frac{V_1}{V_2} = (\frac{a_1}{a_2})^3 = (\frac{1}{2})^3 = \frac{1}{8} \qquad V_2 = 8\ V_2.$

45. Solution: 420.
101 + 103 + 107 + 109 = 420.

46. Solution: 132.
The greatest value of x is 11. $11x + 11 = 11 \times 11 + 11 = 11 \times 12 = 132$.

47. Solution: 25.
The total area of the figure, in square units, is $\frac{5 \times 5}{2} \times 2 = 25$.

48. Solution: 0.
17 = 9 + 8. So one digit must be 9. The probability that none of the digits is 9 will be 0.

49. 34.

Method 1:

By Pythagorean Theorem, $a^2+b^2=c^2 \Rightarrow a^2+10^2=26^2 \Rightarrow a=\sqrt{26^2-10^2}=24$.
The sum of the legs is 10 + 24 = 34.

Method 2:
This is a $5n$–$12n$–$13n$ right triangle. $n = 2$. So the answer is 10 + 24 = 34.

50. Solution: $\frac{1}{24}$.

$\frac{V_1}{V_2}=\frac{l\times w\times h}{3l\times 4w\times 2h}=\frac{1}{24}$.

51. Solution: −1.
Multiplying n by both sides of $n + (1/n) = 5$: $n^2 + 1 = 5n \Rightarrow n^2-5n=-1$.

52. Solution: 0.

By the middle point formula, $x=\frac{x_1+x_2}{2}=\frac{-5+5}{2}=0$.

We know that product is zero without calculating y coordinate.

53. Solution: 24.

$a + b = 16$ (1)
$b + c = 19$ (2)
$a + c = 13$ (3)
(1) + (2) + (3): $2(a + b + c) = 48 \Rightarrow a + b + c = 24$.

54. Solution: 22.
$17 + 17^2 = 17(1 + 17) = 17 \times 18 = 17 \times 2 \times 3 \times 3$.
The sum of the distinct prime divisors is 17 + 2 + 3 = 22.

55. Solution: 9.
$3^n = 27^3$ can be written as $3^n = 3^9$. Thus $n = 9$.

56. Solution: 300°.
The smaller angle is 60° and the larger angle is 360° − 60° = 300°

57. Solution: 14.

$$P=\frac{3}{5}=\frac{\binom{21}{1}}{\binom{21+b}{1}} \quad \Rightarrow \quad \frac{3}{5}=\frac{21}{21+b} \quad b=14$$

58. Solution: 3.

The area of an equilateral triangle is: $A=\frac{\sqrt{3}}{4}a^2$. We know that $a=2\sqrt[4]{3}$.

$$A=\frac{\sqrt{3}}{4}(2\sqrt[4]{3})^2=3.$$

59. Solution: 7.
The sum of the digits must be divisible by 9.
$5 + K + 6$ must be divisible by 9, or $2 + K$ must be divisible by 9. So $K = 7$.

60. Solution: 100 minutes.
James' rate is 1/10 and Evan's rate is 1/20.
Let t be the time they can finish the 15 walls when they work together, we have

$$(\frac{1}{10}+\frac{1}{20})\times t=15 \quad \Rightarrow \quad t=100 \text{ minutes.}$$

61. Solution: 18 minutes.
Let t be the time it takes for them to meet.

$$(\frac{40}{60}+\frac{60}{60})\times t=30 \quad \Rightarrow \quad t=18 \text{ minutes.}$$

62. Solution: 143.
Let two numbers be a and b.
$GCF(a, b) \times LCM(a, b) = a \times b \Rightarrow 7 \times LCM(a, b) = 1001 \Rightarrow LCM(a, b) = 143.$

63. Solution: 56.
We can form a triangle by connecting three points.

$$\binom{8}{3}=\frac{8\times 7\times 6}{3\times 2\times 1}=56.$$

64. Solution: $300.

Let the original price be x.
$50\% \times 50\% \times x = 75 \quad \Rightarrow \quad x = 300.$

65. Solution: 210.
$7 \times 17 + 91 = 210$

66. Solution: 20.
$2 \times \binom{5}{2} = 20.$

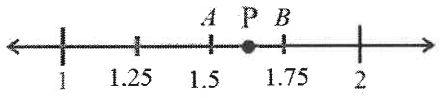
67. Solution: 54.
$6(3 \times 3) = 54.$

68. Solution: 1.
$b = 4, a = 3.$
$b - a = 1.$

69. Solution: 25.
$4x + y = 17$ (1)
$x + 4y = 8$ (2)
(1) + (2): $5x + 5y = 25.$

70. Solution: $5\frac{3}{5}$.
$(2 + 3 + 5 + 7 + 11)/5 = 28/5 = 5\frac{3}{5}.$

71. Solution: 13/8.
$x_P = \frac{x_A + x_B}{2} = \frac{1.5 + 1.75}{2} = 1.625 = 13/8.$



72. Solution: 4.
$1 + 2 + \ldots + 10 + 11 = \frac{(1+11) \times 11}{2} = 66.$

$66 + 4 = 70$ which is divisible by 7. So the answer is 4.

73. Solution: 100.
$50 \times 1.5 = x \times 0.75 \quad \Rightarrow \quad x = 100.$

74. Solution: 2/9.

The sum of two numbers at most can be 12.
Prime numbers greater than 5 are 7 and 11.
$7 = 6 + 1 = 1 + 6 = 5 + 2 = 2 + 5 = 4 + 3 = 3 + 4$.
$11 = 6 + 5 = 5 + 6$.
The probability is $8/36 = 2/9$.

75. Solution: 132.
Let the three sides be $11a$, $11b$, and $11c$, with $11c$ the longest side. The perimeter is $(11a + 11b + 11c) = 11(a + b + c) = 11\ (3 + 4 + 5) = 11 \times 12 = 132$.

76. Solution: 16.
$a+b+c+d+e+f=15\times 6$
$a+b+c+d+e=15\times 6-10=80$.
The answer is $80/5 = 16$.

77. Solution: $\frac{2}{15}$.

$$\frac{5x}{2y}=\frac{5}{6} \qquad \Rightarrow \qquad 3x=y.$$

$$\frac{2x}{5y}=\frac{2x}{5\times 3x}=\frac{2}{15}.$$

78. Solution: 76.
$$\left\lfloor \frac{1000}{13} \right\rfloor = 76.$$

79. Solution: 13.
$221 = 13 \times 17$.
$247 = 13 \times 19$.
The answer is 13.

80. Solution: 7.
$$\sqrt[3]{49} \times \sqrt[6]{49} = 49^{\frac{1}{3}} \times 49^{\frac{1}{6}} = 49^{\frac{1}{3}+\frac{1}{6}} = 49^{\frac{1}{2}} = 7.$$