
MATHCOUNTS

■ Chapter Competition ■
Practice Test 4
Sprint Round Problems 1–30

Name _____

**DO NOT BEGIN UNTIL YOU ARE INSTRUCTED
TO DO SO.**

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

1. Todd had 2016 baseball cards. He sold $\frac{1}{3}$ of them and then gave away $\frac{3}{7}$ of his remaining ones. How many cards does Todd have left?
2. Bob has a bag of marbles. Among them, 35% are green, 20% are yellow, and 80% of the remaining ones are blue. Find the number of marbles in the bag if the number of blue marbles is 36.
3. If 74 hens eat 47 kilograms of wheat and lay 888 eggs in 74 days, what is the average number of eggs produced per hen?
4. Ten containers labeled 1 through 10 in order are placed in a row. The number of marbles in any one container except container 1 is two times the number of marbles in the container before it. If container 9 has 512 marbles and container 10 has 1024 marbles, what is the number of marbles in container 5?
5. The formula $d = 11t^2$ is used to calculate the distance, d , in meter, a free falling object, starting from rest, will travel in t seconds. How many kilometers will the object travel in 11 seconds? Express your answer as a decimal to the nearest thousandth.
6. What is the sum of the reciprocals of the positive integer factors of 24? Express your answer as a common fraction.
7. Point P is on AB . Point A has coordinates $(4, 12)$ and point B has coordinates $(20, 6)$. The ratio of $AP : PB = 2 : 3$. Find the sum of the coordinates of point P .
8. What is the smallest integer whose cube is greater than 30,000?
9. Apples are distributed to 100 kids such that every kid gets at least one apple and each kid gets a different number of apples. Find the least number of apples needed.

10. Chelsea had a bag of balls with half of them red and half of them white. She added 15 red and 3 white balls to the bag. The fractional part of red balls is $\frac{3}{5}$. How many balls are in the bag after addition?

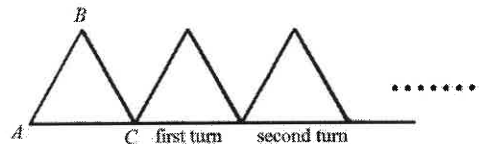
11. Define $m \# n = mn - m - n + 11$ for all positive integers m and n . Find the value of $x + y$ if $x \# y = 21$.

12. What is the difference between the sum of all positive integers from 1 to 100 and the sum of all square numbers from 1 to 100?

13. The sum of a number, x , and its reciprocal is 6. What is the smallest value of x ? Express your answer in simplest radical form.

14. A container has 25 balls green or blue. Alex takes out one half of the green balls and one third of blue balls. Find the greatest number of balls Alex takes out.

15. Triangle ABC is an equilateral triangle with side length of 6. The triangle is turned without sliding along a straight line 756 times as shown in the figure. Find the distance travelled by point A . Express your answer in terms of π .



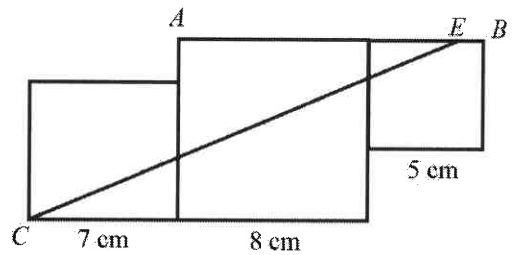
16. Find the probability that a randomly selected positive integer less than or equal to 2016 is divisible by 21? Express your answer as a common fraction.

17. A developer has 100 acres and he would like to divide it into smaller lots. Some should be 30 acres, some should be 15 acres, and some should be 5 acres. If the developer must have at least one lot of each type, how many different ways can he divide up the 100 acres?

18. Find the perimeter of a right triangle whose hypotenuse is 2 and whose area is 1. Express your answer in simplest radical form.

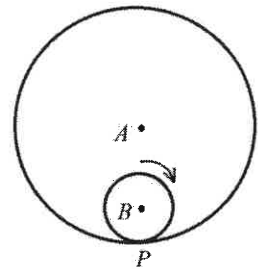
19. A bag of food is enough to feed 16 ducks and 12 chickens for 6 days. The same bag of food can feed 9 ducks and 10 chickens for 8 days. If this bag of food is used to feed only 8 chickens, how many days will it last? Assume that each duck eats at a constant rate every day and each chicken also eats at a different constant rate every day.

20. Three squares of side lengths 7 cm, 8 cm, and 5 cm are put together to form a new figure as shown. Points A , B , and C are vertices of the squares. Line segment CE divides the figure into two parts of the same area. What is the number of centimeter in the length of AE ?

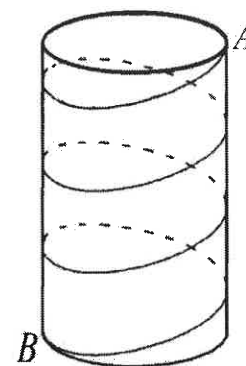


21. A jar contains 7 red, 11 blue and 13 yellow marbles. Blue marbles are then added in order to change the probability of randomly selecting a blue marble from the jar to "greater than $\frac{2}{3}$." What is the least number of blue marbles that must be added?

22. The circumference of circle A is 12π and the circumference of circle B is 4π . Circle B rolls around the inside of circle A without slipping. When P first returns to its original position, what is the number of revolutions it must make?



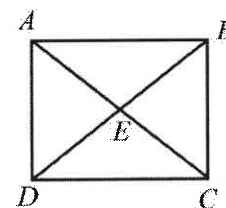
23. A white cylindrical silo has a diameter of $8/\pi$ feet and a height of 21 feet. A wire is painted on the silo, as shown, making three and half revolutions around it. What is the length of the wire in feet? Express your answer in simplest radical form.



24. What is the probability that a randomly selected three-digit whole number is divisible by 3 and the hundreds digit is also 3? Express your answer as a common fraction.

25. Alex rides half of the time at the rate of 9 miles per hour and the rest of the time at the rate of 6 miles per hour for a journey of 90 miles. It takes him x hours to ride the first 45 miles and y hours to ride the second 45 miles. Find the value of $y - x$.

26. How many continuous paths starting from E , along the segments of the figure, and back to E , do not revisit any of the segments?



27. Find B if $\frac{B}{1221} = 0.\overline{3A7}$, where B is a positive integer and A is a digit in the repeating decimal representation.

28. The minute hand of a clock measures 7 cm from its tip to the center of the clock face, and the hour hand from its tip to the center of the clock face is 4 cm. What is the positive difference of the distances, in centimeters, traveled by the tips of both hands in 72 hours? Express your answer in terms of π .

29. The addition problem below has a unique solution. Each of the letters represents a different digit. Find the sum of all possible values of three-digit number DEF if DEF is also a prime number.

$$\begin{array}{r} ABC \\ + DEF \\ \hline DGBD \end{array}$$

30. The sum of 3 positive real numbers is 12 and the sum of their squares is 54. What is the largest possible value for one of the numbers?