MATHCOUNTS

Chapter Competition
Practice Test 3
Sprint Round Problems 1–30

Name ____________________________________

DO NOT BEGIN UNTIL YOU ARE INSTRUCTED TO DO SO.

This round of the competition consists of 30 problems. You will have 40 minutes to complete the problems. You are not allowed to use calculators, books or any other aids during this round. If you are wearing a calculator wrist watch, please give it to your proctor now. Calculations may be done on scratch paper. All answers must be complete, legible and simplified to lowest terms. Record only final answers in the blanks in the right-hand column of the competition booklet. If you complete the problems before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

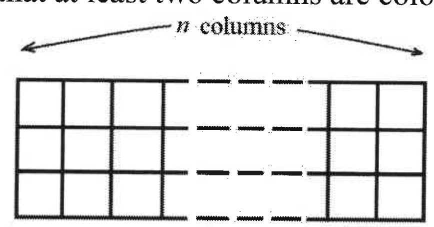
1. Betsy has collected \$6.25 in nickels and dimes. She has exactly 41 dimes. How many nickels does she have?

2. One of the following four-digit numbers is not divisible by 4: 20184, 20174, 20164, 20144, 20104. What is the product of the units digit and the tens digit of that number?

3. The heights for Alex, Bob, Cathy, Debra, and Emma are 147 cm, 149 cm, 150 cm, 151 cm, and 153 cm, respectively. Find Frank's height that is 3 cm less than the average height of five students mentioned before.

4. What is the greatest three-digit number which is a multiple of 17?

5. Each small square in the diagram of 3 by n grid below is colored red (R), yellow (Y) or blue (B). What is the least value of n to guarantee that at least two columns are colored exactly the same way?



6. The numbers 1 through 110 are written on 110 cards with one number on each card. Sally picks one of the 110 cards at random. What is the probability that the number on her card will be a multiple of 5 or 11? Express your answer as a common fraction.

7. Alex is a publisher who published a book a year ago. He paid the author \$20,000. The cost for printing a copy of the book was \$15. He also paid the bookstore 30% of the sale price of \$30 each book. He managed to get 10% profit at the end of the year. Find the least number of copies of the book sold.

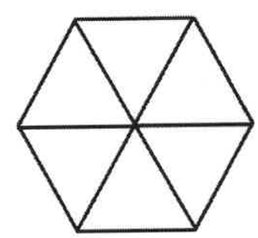
8. Bob received five email messages yesterday in the following order: A, B, C, D, and E. Whenever he opened his mail box, he always replied the most recent message first. Which one of the following is the possible order he replied to the five messages:
(1) ABECD (2) BAECD (3) CEDBA (4) DCABE (5) ECBAD

9. Using each of the digits only once, in how many ways can the digits 1, 3, 5, and 7 be placed, one digit per box, such that 0. □□ > 0. □□.

10. The line $y = 2016 + 504x$ contains points in how many quadrants of the Cartesian coordinate plane?

11. The time is 9:00. The hour hand is pointing west on a 12-hour clock. Which direction (north, south, east or west) is the hour hand pointing 75 hours later?

12. Six equilateral triangles each with side length 1 can form a regular hexagon of the side length 1. How many such equilateral triangles are required to form a regular hexagon of side length 6?



13. Determine the ratio of the surface area of a rectangular prism to its volume if its length, width, and height are the zeroes of the polynomial $x^3 - 3x^2 + 13x - 15 = 0$. Express your answer as a common fraction.

14. Let 7, 11, and c be the lengths of the sides of a triangle. If c is an integer, then what is the difference between the largest and smallest possible value of c?

15. A taxi driver charges the rates as follows:
    First person: \$4 for first mile, \$0.40 for each additional 1/3 mile.
    Additional people: \$3.3 for each additional person.
    If five people share a 32.5-mile ride, how many dollars is the total cost of the ride?

16. How many square units are in the area of the convex quadrilateral with vertices (0, 0), (0, 5), (3, 4), and (7, 0)? Express your answer as a mixed number.

17. Alex and Bob run a lap in 100 seconds and 150 seconds respectively around a 900 m track in opposite directions and starting from the same point. How many meters does Bob need to continue to run to reach the starting point when they meet for the ninth time (not counting the start)?

18. In how many ways can 36 be written as the sum of two primes?

19. Sam uses exactly 100 non-overlapping square tiles, each 1 cm by 1cm, to make four squares. How many ways are there to do so?

20. If we let $(2\sqrt{2}+\sqrt{7})^{2016}=m$, find the expression of $(2\sqrt{2}-\sqrt{7})^{2016}$ in terms of m.

21. There exist positive integers x, y, and z satisfying $29x + 30y + 31z = 366$. Compute the value of $x + y + z$.

22. Gauss Math Club bought some math books for the winners of their school math competitions last year. Table below shows the number of copies of each book they purchased and the total cost.

Purchase date	Books					Total cost
	A	*B*	*C*	*D*	*E*	
Dec. 15, 2014	1	3	4	5	6	$2,016
Mar. 20, 2014	1	5	7	9	11	$3,499

What is the cost to buy one book of each kind?

23. A printer is used to print out the squares of the first 30 positive integers. How many digits are printed by the printer?

24. If the volume of a regular tetrahedron is tripled without changing its height, by what factor is the length of each side increased? Express your answer in simplest radical form.

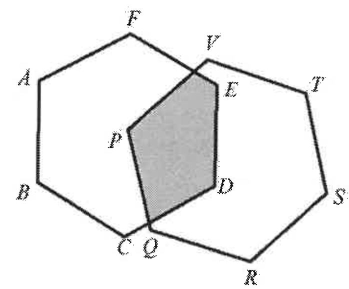
25. Sally has a bag which at the start contains 15 red marbles and 20 white marbles. When she draws out a red marble, she puts the marble back and adds four more red marbles. When she draws out a white marble, she puts the marble back and adds seven more white marbles. What is the fewest number of rounds of draws/replacements/additions after which the bag could contain exactly 75 marbles?

26. What is the sum of all two-digit positive integers that can be written with the digits 1, 2, 3 and 4 if no digit is allowed to use more than one time in each two-digit positive integer?

27. If x and y are positive integers each less than 20 for which $2(x - y)^3 + 4y^2 = 254$, what is the sum of all possible positive integer value for x?

28. When its digits are reversed, a particular positive three-digit integer is increased by 20%. What is the original number?

29. Two congruent regular hexagons *ABCDEF* and *PQRSTV* are placed such that *P* is on the center of the hexagon *ABCDEF* as shown. If the area of each hexagon is 6, find the area of the shaded region.



30. Each day, four out of the six teams in a class are randomly selected to participate in a MATHCOUNTS trial competition. What is the probability that Team *A* is selected at least two of the next four days? Express your answer as a common fraction.